The Application of Whale Optimization Algorithm in Array Antennas

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***Abstract*—With the continuous improvement of various radio system performance indicators, the research work on antenna has become particularly important. According to different scenarios and requirements, practical projects also need the corresponding antennas to produce different radiation patterns. By reasonably setting the parameters of the array antenna, the target radiation pattern can be obtained to meet real life applications. When the array antenna has a large number of basic units and the expected far-field pattern is complicated, the design of the array antenna becomes a complicated optimization problem. To solve this problem, Whale Optimization Algorithm (WOA) is proposed. WOA is not only simple and fast, but can also get the global optimal solution. Therefore, WOA has developed rapidly in recent years. However, the application of this algorithm in the field of antenna design is still relatively rare, thus using WOA to solve the optimization problem of array antenna design is very valuable.**

Keywords-Multi-source Data Fusion; ICP Algorithm; IMU; Three-dimensional Reconstruction

I. INTRODUCTION

The field of antenna design is still relatively rare. There With the continuous improvement of various radio system performance indicators, the research work on antenna has become particularly important. According to different scenarios and requirements, practical projects also need the corresponding antennas to produce different radiation patterns. By reasonably setting the parameters of an array antenna, the target radiation pattern can be obtained to meet real life applications.

In the early stage, analytical methods and traditional engineering optimization methods were used to solve such problems. However, these solutions not only suffer from long calculation time and lack of precision, but were also unable to solve the synthesis of high dimensional complex demand radiation pattern.

With the emergence and rapid development of various intelligent optimization algorithms, array antenna researchers found intelligent optimization algorithms very suitable for solving such complex optimization problems, and started using various intelligent optimization algorithms to solve array antenna optimization problems. As a new intelligent optimization algorithm, WOA has not been widely studied. This paper will study the characteristics of WOA and apply it to the optimization problem of an array antenna. The algorithm mimics the three steps of hunting behavior: encircling prey, spiral bubble-net attacks and searching prey respectively. The three steps of WOA achieve the following functions: first, "encircling prey" enables the whales to swim to the nearest location, which improves the search ability; second, "spiral bubble-net attacks" can improve the convergence speed and local search in a spiral way, which improves the search efficiency of whales; third, "searching prey " is the behavior that whales search for the prey randomly according to the position of each other so as to enhance the global search ability. Since WOA searches globally for optimal solutions, it is considered an effective global optimizer [8]. As a result, WOA has developed rapidly in recent years. However, the application of this algorithm in before, using WOA to solve the optimization problem of array antenna design can be very valuable.

II. BASIC PRINCIPLES OF WOA

A. Encircling Prey

Whales need to determine the target location first, and then surround and hunt. Whale optimization algorithm assumes the current optimal or near-optimal position as the target position. After the optimal candidate solution is established, the positions of other whale individuals are iteratively updated to gradually approach the optimal search for local search. The specific process can be shown in the following formulas (1) and (2):

$$\mathrm{D} = \left| C \cdot X^*(t) - X(t) \right| \tag{1}$$

$$X(t+1) = X^*(t) - A \cdot D \tag{2}$$

Update the positions of the other searches as shown in Equations (3) (4) (5) below:

$$A = 2\alpha \cdot r - \alpha \tag{3}$$

$$C = 2r \tag{4}$$

$$\alpha = 2\left(1 - \frac{t}{\max_t}\right) \tag{5}$$

The above equations are commonly used where $\max_t$ is the maximum number of iterations and α is an important variable of WOA. As shown in the equations, the change in convergence factor α will affect the value of the coefficient vector A and indirectly control the activity of the whale. The shrinking encircling mechanism is achieved through changing the value of convergence factor α $Fig.1$ is a schematic describing the mechanism of encircling prey.

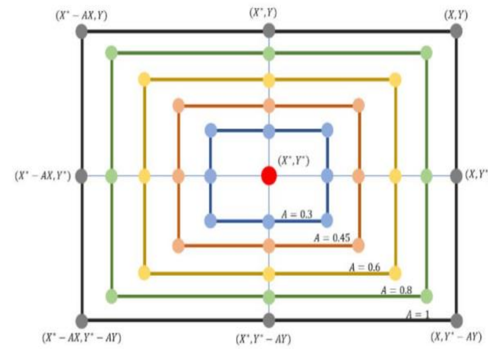

Figure 1. Schematic of encircling prey

B. Spiral Bubble-net Attack

The shrinking encirclement mechanism is that the whale's encirclement of food will gradually shrink in the process of hunting. Whales, while contracting and encircling the food, also swim spirally to the food, which is the spiral position update.

The implementation of the shrinking encircling mechanism mainly depends on changing the value of convergence factor α . When the value of convergence factor α is small, the value of the coefficient vector A will also become smaller, and the value of the coefficient vector A will affect the search ability of search agents. By increasing the value of A, the search range of search agents will grow to a larger range, so that the global search range of the group will be expanded. The global search ability will be enhanced and less likely to be trapped by a local optimum. When the value of A decreases, the search area of search agents will be smaller, hence increasing the local search ability of the group as well as the search speed. Over the course of the entire iteration, α decays linearly from 2 to 0, making A changes within the interval $[-\alpha, \alpha]$. When the value of A is set between $(-1,1)$, it means that the positions of search agents in the next iteration may be anywhere between the current position and the current optimal position. Therefore, search agents are still active in the shrinking bubble-net. When random vector r is between $[0,1]$, the entire process of the shrinking encircling mechanism can be represented by the model shown in $Fig.2$:

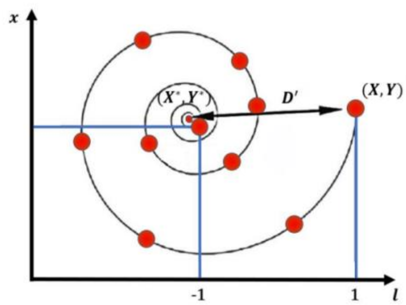

Figure 2. Spiral bubble-net attack of prey

The mechanism of spiral updating position can be represented by the following equation (6):

$$X(t+1) = D' \cdot e^{bl} \cdot \cos(2\pi l) + X(t) \quad (6)$$

Where $D' = \left| X^*(t) - X(t) \right|$ and represents the distance between the optimal location and the current search agent, b is a constant used to define the shape of spiral movement and l is a random variable ranging between $[-1,1]$. When l reaches 1, it indicates that the current whale is the farthest away from the optimal position. When the value of l is -1, it indicates that the current agent is the closest to the optimal position. The symbol "·" is element-by-element multiplication and e represents the base of the natural logarithm.

As can be seen from the figure above, the distance between the current agent and the optimal target position needs to be calculated before the spiral position is updated. The figure above is the mathematical model of motion obtained by simulating the spiral updating position mechanism of humpback whales.

When humpback whales shrink and encircle prey, they also update their spiral positions. It is necessary to assume that each behavior has a certain probability if we were to use a mathematical model to describe these simultaneous behaviors.

The mathematical model is as follows (7):

$$X(t+1) = f(x) = \begin{cases} X(t) - A \cdot D & P < 0.5 \\ D' \cdot e^{bl}\cos(2\pi l) + X(t) & P \geq 0.5 \end{cases} \quad (7)$$

C. Searching Prey (global exploration)

In the actual hunting process of humpback whales, the current searched fish school may not be the optimal fish school in the hunting space. Therefore, humpback whales will also change their positions according to the positions of other whales. As shown in $Fig.3$, global random search is performed for the best fish school in the space. This random search mechanism is simulated by random variables A. In the algorithm, when $0 < |A| < 1$, the whale launches an attack on the prey. When $|A| > 1$, the whales will carry out a global random search for prey, where each humpback whale updates its position according to a search agent randomly selected in the global space. This mechanism increases the population diversity of the algorithm and significantly improves the global search ability of WOA. In the equations below, D is the distance between the position of the current whale and the position of any random search agent, $X_{rand}(t)$ is the position of a random agent in the population. The specific mathematical model is shown in Equations (8) and (9) below:

$$D = \left| C \cdot X_{rand}(t) - X(t) \right| \quad (8)$$

$$X(t+1) = X_{rand}(t) - A \cdot D \quad (9)$$

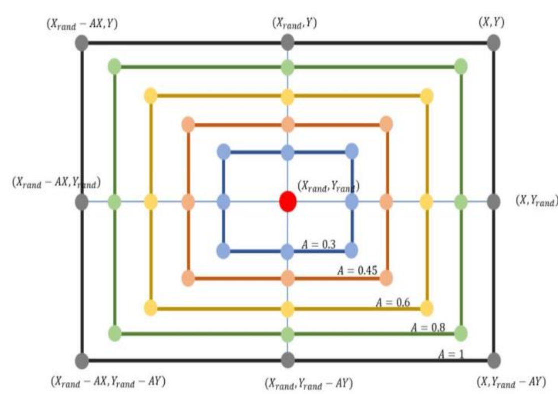

Figure 3. Schematic of searching prey (global exploration)

III. IMPROVING WOA

In the aspect of global search, the WOA algorithm performs global search through the current point and the current optimal point.

A. Logistic Mapping

At present, there are usually many random variables in algorithms based on crowd behavior research. These random variables are generally adjusted through probability. However, these parameters obtained by probability are too random, which are likely to slow the convergence speed of the algorithm and affect the accuracy of the solution. Many researchers now use logistic mapping instead of random probability to solve this problem. Logistic mapping is a typical model for studying the behavior of complex systems such as dynamic systems with discrete time, chaotic and fractal dimensions [4]. It is a nonlinear iterative equation described as follows:

$$x_{n+1} = \mu x_n\left(1-x_n\right),\ \ \mu\in\left(0,4\right),\ \ x_n\in\left[0,1\right] \quad (10)$$

In this equation, n represents the number of iterations, x_n represents the insect-population in the n_{th} generation and $\left(1-x_n\right)$ represents the influence of environmental factors. μ is a system bifurcation control parameter closely related to the dynamic characteristics of chaotic logistic mapping system. Different values of μ will have different effects on the system. When $\mu<1$, this suggests the insect-population decreases and, when $\mu>1$, it means the insect-population increases. The impact of μ on the distribution of logistic mapping is described in $Fig.4$:

where,

a) When $0<\mu\le 1$, no matter the initial value of the system and the number of iterations, the final system trajectory will converge to 0;

b) When $1<\mu\le 3$, there will be two steady-state solutions: 0 and $1-\dfrac{1}{\mu}$, and after a number of iterations, the result will converge to either one of them;

c) When $3<\mu\le 4$, the system will start to exhibit some periodic trajectories;

d) When $3.569945972\le\mu\le 4$, the system is in a chaotic state, where, the result generated by iterations has pseudo-randomness, as well as a strong sensitivity towards the state of the initial value;

e) When $\mu=4$, the distribution of x becomes uneven. A U-shaped relationship is observed with highest frequency at the two extremes.

$Fig.4$ shows the distribution of logistic mapping with different values of μ. It can be seen that as μ is closer to 4, the system becomes more evenly distributed. While when $\mu=4$, the distribution of x is more frequent at the two extremes and less frequent in the middle. This suggests the closer the value of μ is to 4, the better the outcome. Therefore, in this paper, we establish the logistic mapping using $\mu=3.99$. $Fig.5$ is the logistic mapping based on $\mu=3.99$, where T is the number of iterations, the $Y-axis$ represents the value of x.

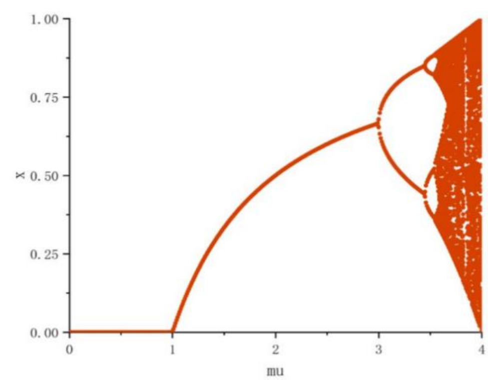

Figure 4. The distribution of logistic mapping with different values of μ

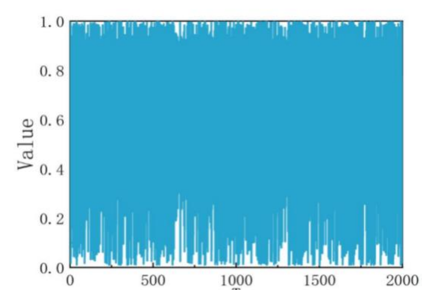

Figure 5. The distribution of logistic mapping when μ = 3.99

B. Inertia Weight

Inertia weight is a concept first appeared in Particle Swarm Optimization (PSO), where the changes of particle coordinates are related to the inertia weight in the iterations of PSO. When the value of inertia weight is large, the step size for the search becomes relatively large, which improves the global search ability of the algorithm. When the value of inertia weight is small, the local search ability of PSO will be better, and the accuracy of the optimal solution will also improve, but the search may be trapped by a local optimum. In this section, we introduce inertia weight into WOA, and applies inertia weight to the two steps: encircling prey and spiral bubble-net attack. Weight is added to the global optimal candidate solution, and the next group of whales will search according to the historical optimal information with added weight [5][8]. This process is updated as equations (11) and (12).

$$x(\mathrm{t}+1)=\omega(t)\cdot x_{*}(t)-A\cdot D \tag{11}$$

$$x(t+1)=D'\cdot e^{bl}\cdot\cos(2\pi l)+x(t)\cdot\omega(t) \tag{12}$$

Where $\omega(t)$ is the inertia weight, which adjusts the step size of the search. According to its characteristics, this paper chooses equation (13) as the iterative update of the inertia weight, where $\mu=t/\max\_t$, $\omega_{\max}=0.9$, and $\omega_{\min}=0.4$.

$$\omega(t)=\omega_{\max}-(\omega_{\max}-\omega_{\min})\cdot 2\mu\cdot sqrt(1-\mu^2) \tag{13}$$

$Fig.$6 Shows the relationship between the inertia weight and the number of iterations $(t\le 1000)$. Because of the fast convergence speed of WOA, it is easy to fall into the local optimum, and it is basically stable at 700 iterations. At this time, the inertia weight increases rapidly, and the step size is expanded again to carry out the global search, and finally the optimal solution is determined. Hence, by introducing the inertia weight, we can better balance the global search and local search ability of WOA.

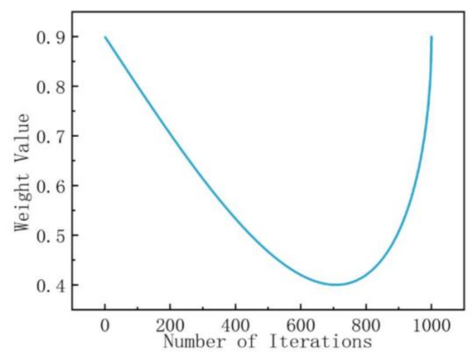

Figure 6. The relationship between the inertia weight and the no. of iterations

IV. CHEBYSHEV PATTERN SYNTHESIS

If the array antenna has a high sidelobe, the strong scattering points at the sidelobe will produce strong reflection of energy. This may cause the radar to mistakenly believe that there is a target in the main lobe direction of the antenna and may miss the target in the main lobe, making the radar to fail to work properly. Therefore, low sidelobe array antennas can not only help radars to perform normal target detection function, but can also improve the battlefield survivability of radars. According to the fundamental theory of antenna array, applying appropriate excitation amplitude on all the basic antenna units will help us obtain lower sidelobe levels. Chebyshev pattern synthesis and Taylor synthesis are methods commonly used in the synthesis of low sidelobe array antenna patterns [1][2][6]. Antenna pattern synthesis is the inverse process of pattern analysis. Pattern synthesis is to calculate the number, position, excitation current amplitude and phase of antenna array elements according to the given pattern conditions (sidelobe level, beam width, pattern shape, etc.). In this section, we will introduce the analytical method used to solve the optimal pattern synthesis of linear array antennas, namely, the Chebyshev pattern synthesis technique. This method solves the contradiction between low side lobe and narrow main lobe of an array antenna. The definition of Chebyshev polynomials (14) is as follows:

$$T_n(x) = \begin{cases} (-1)^n ch(n\ ch|x|) & x < -1 \\ \cos(\text{n} \arccos x) & -1 < x < 1 \\ \text{ch}(\text{n} \arccos x) & x > 1 \end{cases} \quad (14)$$

Let $x = \cos\ \delta$, then $T_n(\cos\ \delta) = \cos(n\delta)$. By using the trigonometric functions, $\cos(n\delta)$ can be expanded into the power polynomial of $\cos(\delta)$. Then substitute $x = \cos\ \delta$ into the equation, we can prove that $\cos(n \arccos x)$ is the power polynomial of x. Chebyshev's recurrence equation is then given by equation (15):

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad (15)$$

Chebyshev polynomials have the following three properties:

i. Even-order polynomial has the characteristic of an even function, the polynomial curve is symmetric about the vertical axis, and that is, when n is even, $T_n(-x) = T_n(x)$. While odd-order polynomial has the characteristic of an odd function, that is, when n is odd, $T_n(-x) = -T_n(x)$.

ii. All the polynomials above pass through point $(1,1)$. When $-1 \le x \le 1$, the value of all polynomials oscillates between -1 and 1, and the absolute value of polynomials are less than or equal to 1.

iii. All the zeros of the above polynomials are located at the interval $-1 \le x \le 1$, and the values of the polynomials at the interval $|x| \le 1$ either monotonically increase or monotonically decrease.

Since all the characteristics of Chebyshev polynomials are consistent with the characteristics required in sidelobe patterns, the array factor can be expressed in the form of Chebyshev polynomials.

Given that the Chebyshev polynomials has only side lobes in the interval $[-1,1]$ and the main lobe is outside this interval, we need the variation range of the matrix factor outside the interval $[-1,1]$, which is given by equation (16) :

$$x = x_0 \cos\left(\frac{\phi}{2}\right) \quad (16)$$

In the above equation, let $f_a(\phi) = T_{n-1}\left(x_0 \cos\frac{\phi}{2}\right)$, then the following equations (17) and (18) can be obtained at the angle of maximum radiation $\theta_0 = 90^0$:

$$\phi = \beta d \cos 90^0 = 0, \quad x = x_0 \cos\left(\frac{\phi}{2}\right) = x_0 \quad (17)$$

$$f_a(\phi = 0) = T_{n-1}(x_0) = R \quad (18)$$

Where R is the true value of the desired sidelobe level SLL, as shown in equation (19). Therefore, according to equation (18) and the value of R, x_0 can be calculated.

$$R = 10^{-SLL/20} \quad (19)$$

The Chebyshev polynomials and the matrix factor have the following correspondence, that is, when x_0 changes to $x_0 \cos\left(\frac{\beta d}{2}\right)$, the true value of the matrix factor is the value of Chebyshev polynomial. The independent variable of Chebyshev polynomial is described as follows:

$$f_a(\phi) = T_{N-1}\left(x_0 \cos\frac{\phi}{2}\right) \quad (20)$$

It can be seen that the independent variable of the matrix factor in the Chebyshev polynomial varies in the range $x_0 \to x_0 \cos\left(\frac{\beta d}{2}\right)$, and its range depends on spacing d and x.

With the given sidelobe level and the number of units, we can use the Chebyshev pattern synthesis method to calculate the excitation current corresponding to the optimal pattern. The comprehensive steps of deriving the excitation current are as follows: step one, get the value of R based on the value obtained by the above Equation (18) and the known sidelobe level, and then calculate x_0. In order to simplify the calculation, the above equation can be converted to equation (21):

$$x_0 = \frac{1}{2}[(R+\sqrt{R^2-1})^{\frac{1}{N-1}} + (R-\sqrt{R^2-1})^{\frac{1}{N-1}}] \quad (21)$$

Step two: derive the excitation current of each basic unit. If the matrix factor is equal to the Chebyshev polynomial of order $N-1$, then the coefficients $\cos\frac{\phi}{2}$ of the same power on both sides of the equation should also be equal, and the excitation current of each basic unit can be calculated.

Step three: calculate the radiation pattern of the array antenna.

V. VERIFICATION OF WOA AND CHEBYSHEV PATTERN SYNTHESIS

The optimal radiation pattern can be obtained by Chebyshev pattern synthesis method. For a given sidelobe level, Chebyshev synthesis method can achieve the narrowest zero-lobe width and main lobe width. For a given zero-lobe width, the Chebyshev synthesis method can obtain the lowest sidelobe level. In order to verify the feasibility of WOA in solving array optimization problems, this section attempts to use the algorithm to solve the optimal radiation pattern of a given array antenna, and compares the optimization results with Chebyshev pattern synthesis method, so that the effectiveness and accuracy of WOA can be verified.

Combining the characteristics of the Dolph-Chebyshev radiation pattern, the following two aspects should be considered when designing the objective function: one is that the main lobe beamwidth should be close to the expected beamwidth, and the other is that the side lobe level should meet the design requirements. We will use the two-mask function to build the target radiation pattern, and the optimal radiation pattern should be between the upper function $Mask_U$ and the lower function $Mask_L$, as shown in $Fig.7$. In the main lobe region, the main lobe width of the actual pattern and the target pattern should be equal. In the sidelobe region, the optimal sidelobe level should be equal to the designed sidelobe level. Therefore, the fitness function in this section can be expressed as equation (22):

$$fitness = \sum_{i=0}^{N} \begin{cases} |f(\theta) - Mask_U(\theta)|, & f(\theta) > Mask_U(\theta) \\ 0, & Mask_U(\theta) > f(\theta) > Mask_L(\theta) \\ |f(\theta) - Mask_L(\theta)|, & f(\theta) < Mask_L(\theta) \end{cases} \quad (22)$$

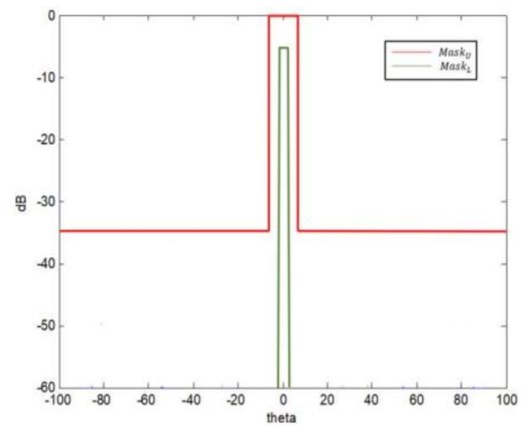

Figure 7. Schematic of two-masks function

Where θ is the radiation angle, $f(\theta)$ is the actual radiation pattern, $Mask_U(\theta)$ is the upper bound of the expected radiation pattern, and $Mask_L(\theta)$ is the lower bound of the objective function. Fitness function is used to represent the error function between the actual pattern and the expected pattern. The smaller the error function is, the closer the actual pattern is to the expected pattern.

In this section, a uniformly arranged linear array of 30 elements is selected as an example. The elements are ideal point source, the spacing between each element is $d = \lambda/2$, and the expected sidelobe level is -35dB. Each element is excited in phase, and the optimization variable is the excitation amplitude I_n of each basic unit. The value of the excitation amplitude is located at the interval $[0,1]$. Since the amplitudes of the 30 array elements are centrally symmetric, the dimension of the optimization variable is half of the array size, which is 15. In this example, the step interval of the radiation pattern is 0.1 degrees, and the angle range of the radiation pattern is $\left[-90^0, 90^0\right]$. According to the equation of uniform linear array, the array radiation pattern $f(\theta)$ corresponding to any optimized variable can be calculated. The selection of the objective evaluation function of the desired radiation pattern is shown in $Fig.4$. Where $Mask_U(\theta)$ and $Mask_L(\theta)$ are the upper and lower bounds of the desired radiation pattern. In this example, the parameters of WOA are set as follows: the number of populations $N = 200$, and the maximum number of iterations is 300.

Therefore, Chebyshev amplitude distribution can be obtained and the radiation pattern of array factors can be calculated. $Fig.8$ shows the comparison of low sidelobe pattern obtained by Chebyshev synthesis and WOA algorithm, and $Fig.9$ compares the amplitudes obtained by Chebyshev pattern synthesis and WOA. As can be seen from the comparison results in $Fig.8$, WOA can obtain a better radiation pattern than Chebyshev synthesis, with the same lobe width and the same amplitude and position of sidelobe levels. $Fig.9$ Compares the excitation amplitudes and it shows that the distribution and trend, as well as the excitation amplitudes obtained by WOA and Chebyshev synthesis are basically the same. The specific differences between the two are shown in Table 1Conclusion.

This paper focuses on the application of WOA in the optimizing the radiation pattern of array antennas. Firstly, WOA is used to solve the amplitude distribution for optimal radiation pattern of uniform linear array with Chebyshev distribution. Then the optimization results of WOA are compared and analyzed with the amplitude distribution obtained by Chebyshev synthesis method, which verifies the effectiveness and accuracy of WOA to solve the optimization problem of array antennas.

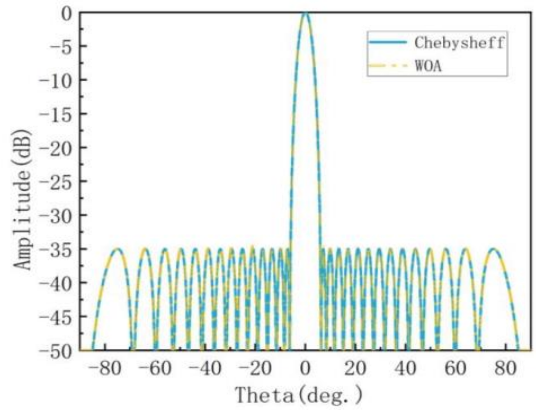

Figure 8. Comparison of low sidelobe radiation pattern obtained by Chebyshev synthesis and WOA

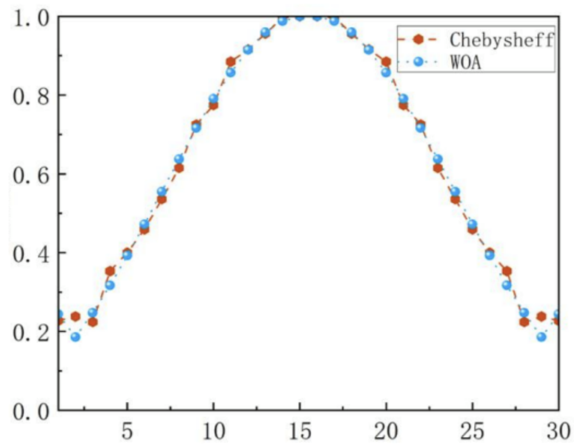

Figure 9. Comparison of distribution of excitation amplitudes obtained by Chebyshev synthesis and WOA

TABLE I. COMPARISON OF OPTIMIZATION RESULTS BETWEEN CHEBYSHEV DISTRIBUTION AND WOA

	amplitude			amplitude	
Element number	Chebyshev	WOA	Element number	Chebyshev	WOA
1	0.2271	0.2307	16	1	1
2	0.238	0.2119	17	0.9965	0.9823
3	0.2241	0.2605	18	0.9561	0.9644
4	0.353	0.2993	19	0.9158	0.8965
5	0.4001	0.3938	20	0.8849	0.8608
6	0.4593	0.4684	21	0.7747	0.7952
7	0.536	0.5472	22	0.7251	0.7009
8	0.6153	0.637	23	0.6153	0.637
9	0.7251	0.7009	24	0.536	0.5472
10	0.7747	0.7952	25	0.4593	0.4684
11	0.8849	0.8608	26	0.4001	0.3938
12	0.9158	0.8965	27	0.353	0.2993
13	0.9561	0.9644	28	0.2241	0.2605
14	0.9965	0.9823	29	0.238	0.2119
15	1	1	30	0.2271	0.2307

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