Abstract—Geometric constraint solving is a hot topic in the constraint design research field. Particle swarm optimization (PSO) is a method to solve the optimization problem from the biological population’s behavior characteristics. PSO is easy to diverge and fall into the local optimum. There are various kinds of improvements. In addition to improving some performance, the corresponding cost is paid. In this paper, a particle swarm optimization algorithm based on the geese is adopted to solve the geometric constraint problem. The algorithm is inspired by the flight characteristics of geese; each particle follows the optimal particle in front of it to keep the diversity; each particle can share more useful information of other particles, which strengthens cooperation and competition between particles. The algorithm balances the contradiction between the search speed and the accuracy of the algorithm to a certain extent. Experimental results show that the proposed algorithm can improve the efficiency and convergence of geometric constraint solving.

Keywords—PSO; Geometric constraint solving; geese; individual extreme; global extreme

I. INTRODUCTION

With the development of computer technology, CAD/CAM has been developed rapidly. The development and application level of CAD/CAM has become an important sign of the national modernization level. Geometric constraint solving is a hot topic in the constraint design research field. A constraint describes a contented relationship. If users have defined a series of relationships, the system will automatically choose the appropriate state to satisfy the constraints after the parameters are modified. This method is called constraint model. Now many scholars study deeply the constraint solving by using numerical calculation theory, artificial intelligence theory, graph theory, freedom analysis theory. There are the integrated solution, the sparse matrix, connection analysis, protocol construction, constraint propagation, symbolic algebra and auxiliary line [1].

Particle swarm optimization (PSO) is a method to solve the optimization problem from the biological population’s behavior characteristics. PSO is easy to diverge and fall into the local optimum. There are various kinds of improvements. In addition to improving some performance [2], the corresponding cost is paid. In view of the above shorts, according to the flight characteristics of geese, the paper proposed two improvements: firstly, global extreme value is transformed into individual extreme of the anterior superior particle value according to historical optimum fitness sorting. So all particles do not direct the same solution, which can avoid the same, maintain diversity, and expand the search scope; secondly, each particle can use more other particles’ useful information to strengthen the cooperation and competition between particles by the individual extreme value weighted mean.

II. PARTICLE SWARM OPTIMIZATION BASED ON THE GEESE

The standard PSO and various improved algorithms focus on how to make the particle swarm more effectively search the optimal solution in the solution space. But in the latter period of the search, particles tend to be identical, and this unification limits the search range of particles. To expand the search range, the number of particles must be increased in the particle swarm, or the particle's pursuit to the global optimum is weakened. Increasing the number of particles will lead to higher computational complexity of the algorithm. Reducing the particle’s pursuit of the global optimum has the disadvantage that the algorithm is not easy to converge. The following improvements can be made to PSO [3]:

A. Improve PSO by using the flight characteristics of geese

In nature, the flight mode of geese is very efficient, and the flight distance of geese increases 72% more than solo goose. In flight of geese, leader goose flaps wings to produce vortex, and trailing companions can assist to fly. So the leader goose is the most laborious. The leader goose is the strongest goose and the other geese lines up in turn. Reference to the inspiration of the geese flight, the strength level of a goose can be regarded as the degree which particle is good or bad, namely historical optimal fitness value of particle. So all the particles will be sorted by the historical optimal fitness value, select the best fitness value optimal particle as the leader goose. The best fitness value of each particle is updated after each iteration, and then all particles are reordered.

The geese line up from front to back according to the historical optimum fitness value, each goose behind only follows its front the better goose flight. That is to say, the anterior goose's individual extreme is the global extreme of the behind goose ($p_{l1}$ replaces $p_{l0}$), and the global extreme
of the leader goose is still its own individual extreme. This is the improvement of the global extreme value of PSO according to the flight characteristics of the geese. Speed formula is updated to:

\[ v_{id}^{k+1} = \omega v_{id}^k + c_1 \times \text{rand}() \times (p_{id} - x_{id}^k) + c_2 \times \text{rand}() \times (p_{(i-1)d} - x_{id}^k) \]  

(1)

The advantages that the front optimal particle individual extreme replaces the global extreme value: All particles fly in more than one direction, avoid the tendency of particles to be identical, maintain the diversity of particles, and expand the search scope; but weakening the particles’ chase to the global extreme lets algorithm not easy to converge.

In the geese flight, geese can push and cooperate with each other through the tail vortex, which is efficient because of group cooperation. The purpose of group cooperation is [4]: firstly, each individual can help other members of the group in the process of growing up; secondly, group cooperation can improve efficiency. In other words, each individual can provide information to the community, and each individual can assist other individuals in searching, just as multiple intelligences collaboration and competition. E.O.Wilson [5] argues that, at least in theory, in the process of mass search for food, each individual in the group can benefit from the new discovery of the group and the experience of all other individuals in the group. In the flight of geese, we think that the leader goose only relies on its own experience to make decisions. The behind geese not only rely on their own experience but also learn from other geese’s experience, and its current value is a reference to the weight, the current fitness value represents the current state. So each goose individual extreme value except the leader goose is transformed to the weighted average value of individual extreme value and its present fitness value f(Xi).

\[ P_a = \frac{\sum_{i=1}^{N} X_i \times P_i}{\sum_{i=1}^{N} X_i} \]  

(2)

Improving \( p_{ad} \) to \( p_{ad} \) has the following advantages: particles use more information to make their own decisions, which makes the algorithm to further reduce the probability of falling into local optimal; individual obtains more incentive, strengthen the cooperation and competition between particles, and accelerate the convergence speed.

Combination with the above two improvements, the speed and position formula of the new algorithm is updated as follows:

\[ v_{id}^{k+1} = \omega v_{id}^k + c_1 \times \text{rand}() \times (p_{id} - x_{id}^k) + c_2 \times \text{rand}() \times (p_{(i-1)d} - x_{id}^k) \]  

(3)

\[ x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \]  

(4)

The new algorithm refers the characteristics of the geese flight. On the one hand, the front optimal particle individual extreme replaces the global extreme value, so all particles fly in more than one direction, avoid the tendency of particles to be identical, and maintain the diversity of particles; On the other hand, the new algorithm makes each particle use more useful information of other particles, replaces the individual extreme with the weighted average value of individual extreme value and its present fitness value. Individual incentives become larger. The algorithm strengthens the cooperation and competition between particles. The combination of the two improvements balances the contradiction between the algorithm search speed and the algorithm accuracy.

B. Steps of GeesePSO

1) Initialize the particle swarm: give population size \( M \), the solution space dimension \( N \), randomly generate the location of each particle \( X_i \), speed \( V_i \).

2) Calculate the current fitness value of each particle with the benchmark function \( f(X) \).

3) Update individual extreme: evaluate the individual extreme value of each particle, compare the current value of the ith particle \( f(X_i) \) with the fitness value of the particle individual extreme value \( P_i \) . If the former is excellent, update \( P_i \), otherwise \( P_i \) is unchanged.

4) Particle swarm sort: all particles are sorted according to the historical optimal value (\( P_i \), fitness value), select the best history optimal fitness value particle as the leader goose, other geese turn back in turn.

5) Calculate the new individual extreme: the leader goose’s individual extreme remains unchanged, calculate the new individual extreme (\( P_a \)) of other particles with the formula (2).

6) Calculate the new global extreme: the leader goose’s global extreme remains unchanged, each goose behind takes the individual extreme of the front superior goose as its global extreme.

7) Update speed and position: update the velocity (\( V_i \)) and position (\( X_i \)) of each particle by formula (3) and (4).

8) Check whether the stopping condition (maximum iteration algebra or minimum error threshold) is satisfied: if it is satisfied, exit; otherwise, go to step (2).

III. GEOMETRIC CONSTRAINT SOLVING

From the point of view of artificial intelligence [6-7], the design problem is essentially a constraint satisfaction problem. Among the many design constraints, geometric constraint is the most basic. It is the basis for expressing other design constraints, and also a priority problem in constraint management and solution technology. The ultimate goal of solving a geometric constraint problem is to determine the specific coordinate position of each geometry in geometry. If the degree of edge generate (DEG) of a Geometry is less than its degree of freedom (DOF), the geometry can be determined by the location of the geometry in which it is bound.

In engineering applications, most mechanical designs come from sketches and existing graphics. In sketch design, the user initially does not care about the exact size of the
graph, but roughly outlines the general shape of the part. The user may make minor improvements on the basis of the existing graphics. Size adjustment is very common, because size can determine the geometry of the parts. Size changes can produce different geometric shapes. The traditional interactive mapping method can give full play to the designer’s ability. But after graphic production, it is difficult to adjust the size because it has no inheritance.

For the constraint problem, it can be formalized as \( (E, C) \) [8] \( (E= (e_1, e_2, \ldots, e_n)) \), it represents geometric elements such as dots, lines, circles, etc.; \( C= (c_1, c_2, \ldots, c_m) \), \( c_i \) represents the constraint between these geometric elements. Since a constraint corresponds to an algebraic equation, the constraint can be expressed as

\[
\begin{align*}
&f_1(x_0, x_1, x_2, \ldots, x_n) = 0 \\
&\ldots \\
&f_m(x_0, x_1, x_2, \ldots, x_n) = 0 \\
&X= (x_0, x_1, \ldots, x_n)
\end{align*}
\]

\( x_i \) is some parameters of the geometric element(ei), for example the two dimensional point can be expressed as \( (x_1, x_2) \). Constraint solving is to find the X formula (5).

\[
F(X)=\sum_{i=1}^{m}\left| f_i \right|
\]

If \( X_j \) satisfies \( F(X_j)=0 \), the \( X_j \) satisfies the formula (1). The constraint solving problem can be translated into an optimization problem. Only \( \min(F(X_j))<\varepsilon \) is required, \( \varepsilon \) is a threshold. To improve the speed of the algorithm, we use the absolute value sum of the \( f_i \) instead of the squares sum to represent the constraint equations. By formula (6) and using the GeesePSO to solve \( \min(F(X_j))<\varepsilon \) ( \( m=n \) is not required), the method can obviously solve the under- constraint and over-constrained problems.

IV. EXPERIMENTAL RESULTS

Figure 1 is the original design. Figure 2 is the new graphic that uses the GeesePSO after part of the size or angle are changed. From the diagrams, the user can modify the size value, and the system of cell membrane optimization algorithm updates graphics in real time according to the new size. That can easily create series parts and modify graphics. According to the above sketch, we compare the genetic algorithm PSO and GeesePSO.

**TABLE I. COMPARISON OF THE EXPERIMENTAL RESULTS OF PSO AND GEEPSO**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Iterations</th>
<th>CPU occupancy time</th>
<th>Iteration Number of the best solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>80</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>GeesePSO</td>
<td>40</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>

It can be seen from table 1, that the GeesePSO is used to solve the geometric constraint problem, and the algorithm can achieve better performance and better convergence than the other algorithms. The new algorithm not only has higher search speed, but also has higher convergence precision. It can balance the contradiction between the search speed and the accuracy.

V. CONCLUSION

Geometric constraint solving is the core of parametric design. The quality of geometric constraint solving is the key to the parametric design system. In this paper, the constraint equations of geometric constraint problems are transformed into optimization models, the problem of constraint solving is translated into optimization problems. As one of the representative methods of swarm intelligence, particle swarm optimization algorithm provides a new solution for nonlinear, non-differentiable and multi-peak complex optimization problems, but it is easy to fall into local optimum and divergence. In this paper, an improved PSO algorithm is proposed by referring to the flight characteristics of the geese.

On the one hand, the global extreme value transforms to the individual extreme of the front optimal particle, all particles fly in more than one direction. That avoids the tendency of particles to be identical, and maintains the diversity of particles. On the other hand, the new algorithm makes each particle use more useful information of other particles, replaces the individual extreme with the weighted average value of individual extreme value and its present fitness value. Individual incentives become larger. The algorithm strengthens the cooperation and competition between particles. The combination of the two improvements balances the contradiction between the algorithm search speed and the algorithm accuracy. Experimental results show that GeesePSO has higher convergence accuracy, faster convergence speed, better global search capability, and a proper balance between detection and development capabilities in geometric constraint solving.

REFERENCES

Figure 1. Original design

Figure 2. New graphics designed by GeesePSO algorithm