

Discussion on the Factors of Stability for Benchmark Example with a Spherical Failure Surface in Clay

Jinhui Liu

College of traffic and civil engineering
Shandong Jiaotong University
Jinan, China
Liumengdi1996@163.com

Wantao Ding*

Geotechnical & Structural Engineering Research Center
Shandong University
Jinan, China
dingwantao@sdu.edu.cn
*The corresponding author

Abstract—A three-dimensional slope stability problem involving a spherical failure surface in clay is often used in the literature as a benchmark example against which numerical models are validated. In the existing research literature, the analytical expression has been obtained for the factors of safety by assuming plane-strain mechanisms during slope failure. And the hypothesis does not comply with the actual project due to the size effect of slope and surrounding constraints placed on the slope. This paper compares and analyzes the results of the existing research literature. And the three analytical expressions under the three kinds of rotational model have been given for the factors of safety. In practice, the value of factor of stability obtained by numerical model should be in a range which is determined by rotational model of failure body. In this paper, when subtended angle is equal to 60° , the value of factor of stability obtained by numerical model for benchmark example should be reasonable in the range from 1.191 to 1.588.

Keywords—Three-dimensional slope stability analysis; Factor of safety; Spherical failure surface; Numerical model; Plane-strain mechanisms; Subtended angle

I. INTRODUCTION

Because of occupying a certain space, failure surface of slope has obvious three-dimensional feature. However, its stability analyses have usually been carried out using a two-dimensional approach. Such failure modes yield conservative estimates of the slope safety when compared with three-dimensional failure patterns. Since the mid 1970s, increasing attention has been directed toward the implementation of three-dimensional stability models [1,2]. Three-dimensional analyses of slopes can be grouped into three categories: the extension of traditional slice methods; numerical approaches, such as the finite element method or the discrete element method; and limit analysis (the plasticity approach). The reader will find a review of the first two categories in a recent article by Griffiths & Marquez [3]. The application of limit analysis to earth slopes started with a paper by Drucker & Prager [4], who applied the kinematic approach of limit analysis to the stability of slopes undergoing plane-strain failure. Where the limit equilibrium methods of columns are most popular and are considered to be the most feasible for

practical problems [5-9]. L.Z. Wu. et al. conducted a series of physical tests, which were conducted to simulate rain-induced slope failure[10]. Jin Man Kim proposed the reliability approach to analyze slope stability with spatially correlated soil properties[11]. A. Johari used the jointly distributed random variables method to assess the reliability of infinite slope stability[12]. Seboong Oh present two case studies of rainfall-induced failure of engineered slopes[13]. Joshua A. White assessed slope stability using stochastic rainfall simulation[14]. L.L.Zhang reviewed the stability analysis of rainfall-induced slope failure[15]. In reality, under rainfall conditions, the degree of saturation within a slope and along a failure surface could be highly variable and pore water pressure could be negative[16]. In recent years, slope-stability analyses have been expanded to include coupled hydromechanical processes under variably saturated conditions[17-18].

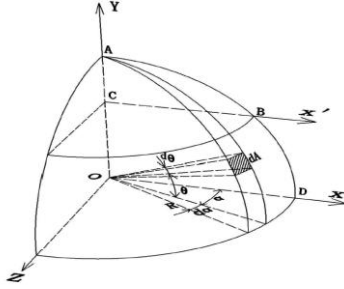
Due to the complexity of soil, the analytical solution of some actual problems can't be obtained. Numerical methods known as a viable option need to be validated using a comparison with a particular problem that involves a spherical failure surface in clay for which a closed-form solution appeared to be available. However, after reconsideration of this benchmark example, it was found that the reported value of the factor of safety obtained from the closed-form solution was several problems. One is the issue of the coordinate system used; the other is the issue of the rotational problems of failure body. By analyzing the existing research methods and considering the effect of slope spatial dimensions during the slope failure, this paper discusses the potential sliding type of failure surface in clay and obtains the factors of safety under different sliding type.

II. ANALYSIS

A. Arm of Resisting Moment

When calculating the resisting moment M_ρ^o , Silvestri [19] considers the moment arm about the axis OA (z-axis, Fig .1) is defined as

$$L_1 = R \cos \psi \quad (1)$$



(b) Coordinate system

Figure 2. Spherical cap failure surface: (a) cross-section of mechanism; (b) coordinate system

Because of occupying a certain space, the failure evolution process of slope has the obvious spatial characteristics. When assuming that the size of purely cohesive slope tends to infinity along the longitudinal and transverse section, the failure mode of slope can be considered plane-strain mechanisms. However, the plane-strain mechanisms of failure is more difficult to be met in the actual engineering, the width and length of the actual slope are limited. So the failure model of slope does not satisfy the plane-strain mechanisms. In the past, considering the plane-strain mechanisms, the hypothesis that the mechanism of failure consists of a rigid body rotation about the axis OA (z-axis, Fig.1) is usually proposed when analyzing the slope stability. Considering the size effect of slope and surrounding constraints placed on the slope, there may be a variety of rotational forms. Analyzing the evolution of slope slip, there are the other two kinds of rotational form. One is that all of the infinitesimal area elements rotate about point O (eg. Moment arm L_0 in Fig.1) ; the other is that all of the infinitesimal area elements rotate about the axis parallel to z-axis (eg. moment arm L_1 in Fig.1).

C. All of the Infinitesimal Area Elements Rotation about Point O

In this rotational form, as shown in Fig .1, the infinitesimal area element dA can be express as eq.(5). Where $dl = R \cos \theta d\psi$ and $ds = R d\theta$, then eq.(5) reduces to

$$dA = R^2 \cos \theta d\psi d\theta \quad (10)$$

The moment arm rotation about point O is given by

$$L_0 = R \quad (11)$$

And the resisting moment M_r^o is given by the following relationship

$$M_r^o = 4S_u R^3 \frac{\pi}{2} (1 - \sin \delta) \quad (12)$$

As a consequence, the factor of safety F is

$$F = \frac{M_r^o}{M_d^o} = \frac{16S_u}{\gamma R \pi \sin \beta} \frac{\pi(1 - \sin \delta)}{2 \cos^4 \delta} \quad (13)$$

D. All of the Infinitesimal Area Elements Rotation about the Axis Parallel to Z-axis

In this rotational form, the infinitesimal area element dA can be express as eq.(10). The moment arm rotation about the axis parallel to z-axis is given by

$$L_1 = R \sin \theta \quad (14)$$

And the resisting moment M_r^o is given by the following relationship

$$M_r^o = 4S_u R^3 \frac{\pi(1 - \sin^2 \delta)}{4} \quad (15)$$

As a consequence, the factor of safety F is

$$F = \frac{M_r^o}{M_d^o} = \frac{16S_u}{\pi \gamma R \sin \beta} \frac{\pi}{4 \cos^2 \delta} \quad (16)$$

III. COMPARISON AND APPLICATION

The geometry and parameters of the benchmark example used in the evaluation of the numerical approaches are (Hung et al.1989; Lam and Fredlund 1993): $\beta = 26.6^\circ$, $\Theta = 60^\circ = \pi/3$, $S_u/\gamma R = 0.1$, $\delta = \pi/2 - \Theta$. In which Θ represents the subtended angle, as shown in Fig .1(a).

The range of subtended angle Θ is between 0° and 90° . Changing the value of subtended angle at intervals of 5 degree, the factors of safety rotation about axis can be obtained using the eq.(9), eq.(13) and eq.(16). The relations between the factors of safety and subtended angles are shown in Fig .3.

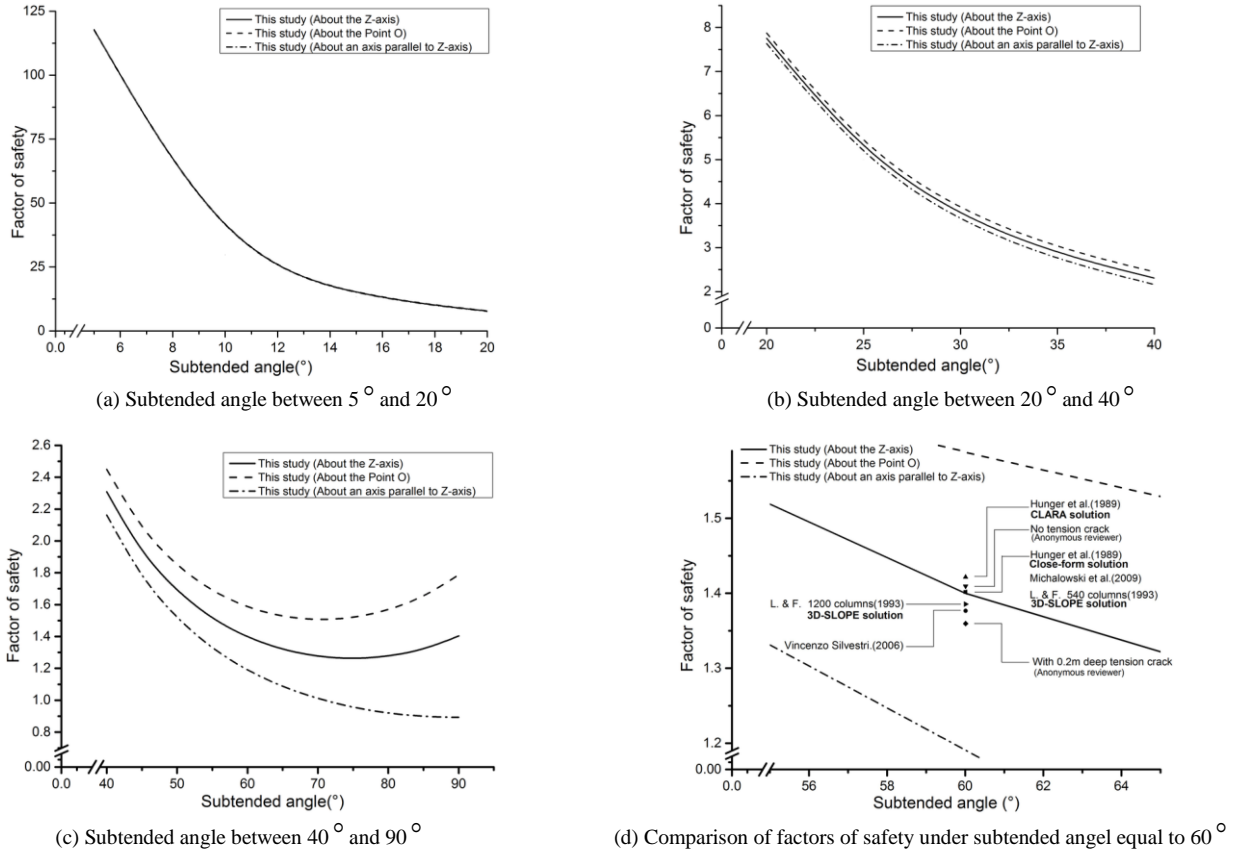


Figure 3. Factor of safety as function of subtended angle for benchmark example

As shown in Fig .3(a) and Fig .3(b), the factors of safety decrease gradually with subtended angles increasing. And the factor of safety rotation about point O is the largest. That rotation about axis parallel to z-axis is the least and that rotation about z-axis is the middle. As shown in Fig .3(c), the factor of safety rotation about axis parallel to z-axis decrease gradually with subtended angles increasing, but the other two factors of safety decrease gradually with subtended angles increasing from 40° to about 75° , when the subtended angles large than 75° , both of the factors of safety increase gradually.

As shown in Fig .3(d), Hungry et al. mentioned that while the closed-form solution of Baligh and Azzouz yielded $F = 1.402$, the numerical model CLARA resulted in $F = 1.422$. Considering a kinematically admissible rotational mechanism in cohesive soils (undrained behavior), Michalowski et al. yield $F = 1.402$. One of the two anonymous reviewers of the present paper obtained $F = 1.41$ using Bishop's simplified method, as implemented in the latest version of the program CLARA. However, the program indicated negative normal stresses in 15% of the weight. With a vertical, planar dry tension crack 0.2m deep, the factor of safety reduced to 1.36, with the negative stresses affecting only 9% of the slide. In addition, Lam and

Fredlund, using the 3D-SLOPE model, obtained $F = 1.402$ when the slope was discretized 540 columns and $F = 1.386$ for 1200 columns. The latter result puzzled Lam and Fredlund because it was lower than the so-called exact value of 1.402 when the number of columns was increased. However, in comparison with the value of $F = 1.191$ obtained in this study(rotation about an axis parallel to z-axis), the factor of safety proposed by Lam and Fredlund, that is, $F = 1.386$, is reasonable. Silvestri obtained $F = 1.377$, the value is in inaccurate. In this study, three factors of safety were proposed: one is $F = 1.4$ (rotation about z-axis), the other is $F = 1.588$ (rotation about point O) and the third is $F = 1.191$ (rotation about axis parallel to z-axis). In past, the so-called exact value of F was considered equal to 1.402 because of assuming plane-strain mechanisms during slope failure. In the actual project, the slope failure does not satisfy plane-strain mechanisms completely, so the value of factor of stability for benchmark example using numerical models should be reasonable in the range from 1.191 to 1.588.

IV. CONCLUSION

A benchmark example often used to validate numerical models for the analysis of three-dimensional slope stability problems, was re-analyzed, and the analytical expressions under three kinds of rotational models have been obtained for the factors of safety. Assuming plane-strain mechanisms during slope failure, the factor of safety should be determined by eq.(9). But in the actual project, because of the size effect of slope and surrounding constraints placed on the slope, the three-dimensional numerical approaches yield solutions should be in a range between the value of factor of safety obtained by eq.(16) and that obtained by eq.(13).

ACKNOWLEDGMENT

The authors acknowledge the financial support provided by the National Natural Science Foundation of China (Grant Nos. 41572275) and the Shandong Province Natural Science Fund (Grant Nos. ZR2012EEM006).

REFERENCE

- [1] Baligh, M. M. & Azzouz, A. S.. End effects on stability of cohesive slopes. ASCE J. Geotech. Engng Div.101, No.GT11, 1975, pp.1105-1117.
- [2] Hovland, H.J. Three-dimensional slope stability analysis method. ASCE Journal of the Geotechnical Engineering Division, 103(9),1977,pp.971-986.
- [3] Griffiths, D.V. & Marquez, R.M. Three-dimensional slope stability analysis by elasto-plastic finite elements. Géotechnique 57, No.6,2007,pp. 537-546.
- [4] Chen, R.H., and Chameau, J.L.Three-dimensional limit equilibrium analysis of slopes. Géotechnique,32(1),1982,pp.31-40.
- [5] Cavounidis,S. On the ratio of factor of safety in slope stability analyses. Géotechnique, 37(1),1987,pp.207-210.
- [6] Hungr, O. An extension of Bishop's simplified method of slope stability analysis to three dimensions. Géotechnique, 37(1), 1987, pp.113-117.
- [7] Gens,A., Hutchison, J.N., and Cavounidis, S. Three-dimensional analysis of slopes in cohesive soils. Géotechnique, 38(1), 1988, pp.1-23.
- [8] Hungr, O., Salgado, F.M., and Byrne, P.M. Evaluation of a three-dimensional method of slope stability analysis. Candian Geotechnical Journal, 26(4),1989,pp.679-686.
- [9] Michalowski, R.L., and Drescher,A. Three-dimensional stability of slopes and excavations. Géotechnique, 59(10), 2009, pp.839-850.
- [10] L.Z.Wu, R.Q.Huang,Q. Xu,et al. Analysis of physical testing of rainfall-induced soil slope failures. Environ Earth Sci 73,2015,pp.8519-8513.
- [11] Jin Man Kim, Nicholas Sitar. Reliability approach to slope stability analysis with spatially correlated soil properties. Soils and Foundations,53(1),2013,pp.1-10.
- [12] A Johari, A.A. Javadi. Reliability assessment of infinite slope stability using the jointly distributed random variables method. Scientia Iranica A, 19(3),2012,pp.423-429.
- [13] Seboong Oh, Ning Lu.Slope stability analysis under unsaturated conditions: Case studies of rainfall-induced failure of cut slopes. Engineering Geology, 184,2015,pp.96-103.
- [14] Joshua A. White, Dashi I. Singham. Slope stability assessment using stochastic rainfall simulation. International Conference on Computational Science, ICCS 2012,pp.699-706.
- [15] L.L.Zhang, J.Zhang, L.M. Zhang et al. Stability analysis of rainfall-induced slope failure:a review. Geotechnical Engineering, Issue GE5,V(164),2011,pp.299-316.
- [16] Buscarnera,G., Whittle, A. Constitutive modelling approach for evaluating the triggering of flow slides. Can. Geotech. J.49(5),2012,pp.499-511.
- [17] Lu, N., Kaya, M. A drying cake method for measuring suction stress characteristic curve, soil-water retention, and hydraulic conductivity function. Geotech. Test.J. 36,2012,pp.1-19.
- [18] Lu, N., Wayllace, A., Oh, S. infiltration-induced seasonally reactivated instability of a highway embankment near the Eisenhower Tunne, Colorado, USA. Eng. Geol.162,2013,pp.22-32.
- [19] Vincenzo Silvestri. A three-dimensional slope stability problem in clay.Can. Geotech. J. 43,2006,pp.224-228.