# An Improved Universal Evidence Combination Method

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**Abstract.** The Dempster-Shafer evidence combination method will appear inconsistent conclusions for the conflict evidence. One new universal evidence combination method was proposed. According to the concept of the Pearson correlation coefficient. Evidence distances which represent the conflict degree were calculated, and then the weight coefficient were further converted. The evidences probability were redistributed by using the weight coefficient. Finally, the improved synthesis rule was used to the evidence combination. The examples results show that the algorithm has strong versatility and stability in the evidences combination, can be applied to information fusion, uncertain information decision-making and other fields.

**Keywords:** Evidence combination, conflict , weight , evidence pretreatment

## 1. Introductiong

DS evidence theory is first proposed by Dempster in the 1960s and further developed by Shafer, and is a theory of reasoning belonging to artificial intelligence areas. As a method of uncertainty reasoning, DS evidence theory provide a natural and powerful way for the expression and synthesis of uncertain information, within the framework of the basic assumptions for the power set of the collection, it can make full use of the probability distribution function, likelihood function and other functions to describe and deal with uncertain information, so get a wide range of applications in information fusion and uncertain reasoning [1-3].

When a large inconsistency or conflict between the evidences, DS evidence combination method can not be used or the results obtained from which may be odds with reality. How to achieve effective multi-source information fusion on high degree of conflicting evidences is the current hot research, a variety of conflicting evidences combination methods are proposed [4-9]. By studying them, the overall synthesis rules can be divided into two broad categories: modified the rules of synthesis methods and the ways to modify the original evidences.

Single conflict evidence synthesis method has its limitations, so, this paper presents one evidence combination method by the fusion of pre-integration rules and synthetic methods modified. First, by converting the Pearson coefficient into weights of evidence, and thus be used to modify the source; and then the modified rules are further used for combination. Experimental results show that the new method can quickly get the proper results of the normal or conflict evidences, and has good adaptability, reliability and fast convergence rate.

## 2. Ds Theory

In the DS evidence theory [9,10], the identification framework Θ of fusion system contain N complete incompatible assumption propositions, its power set P(Θ)=2Θ={A1, A2, A3⋯A2N}.. Evidences E1, E2, E3。。。En. The basic probability distribution functions respectively are m1, m2, m3⋯mn.

$$\sum_{A\in P(\Theta)} m(A)=1 , \quad m(\Phi)=0 , \quad m(A)\geq 0 \tag{1}$$

The corresponding Dempster combination rule is:

$$\begin{cases} m(A)=\dfrac{1}{1-k}\sum\limits_{\cap A_j=A}\prod\limits_{1\leq i\leq N} m_i(A_i) \quad for\ A\neq\Phi \\ \qquad m(\Phi)=0 \end{cases} \tag{2}$$

Where, k is the conflict factor, which reflects the conflict degree between the evidences.

$$k=\sum_{\cap A_j=\Phi}\prod_{1\leq i\leq N} m_i(A_i) \tag{3}$$

In the Dempster combination rule, k is a coefficient measuring the conflict level between the evidences. If k = 1, we can not use the Dempster combination rule for information fusion; and when k → 1, the combination results of a high degree conflict evidences is inconsistent with the actual processing. Conflict is also a kind of information, the extraction and analysis of them, and being added into the combination rules, one new combination method can be got.

## 3. New Combination Method

New method fuses evidence pre-processed and synthetic rules modified. According to the principles of conflict information available, weights of evidences can be obtained from the Pearson coefficient, the more collision with other evidences, the smaller its weight. Then the modified combination rules are used to fuse weighted evidences.

### *3.1 Pearson Correlation Coefficient*

Pearson correlation, also known as product-moment correlation (or product-moment correlation) is a linear correlation proposed by Pearson of British statistician in the 20th century [11]. Suppose there are two variables X, Y, and then the Pearson correlation coefficient of the two variables can be calculated by the following formula:

$$\rho_{X,Y}=\frac{\mathrm{cov}(X,Y)}{\sigma_X\sigma_Y}=\frac{E((X-\mu_X)(Y-\mu_Y))}{\sigma_X\sigma_Y} \tag{4}$$

Where E is the mathematical expectation, cov is the covariance.

$$\mu_X=E(X) \tag{5}$$

$$\sigma_X^2=E(X^2)-E^2(X) \tag{6}$$

Pearson correlation coefficient is one measurement of the degree between two variables. Its value is in a range from -1 to 1, where 1 indicates perfect positive correlation, 0 independent, and -1 indicates a perfect negative correlation.

### 3.2 Weight

Step1: According to (4), calculate the Pearson coefficient p(E1, E2) between the evidence E1, E2;

Step2: The sum of the Pearson coefficient of the evidence Ei is defined as:

$$\delta(E_i) = \sum_{j=1, j \neq i}^{n} p(E_{i,} E_j) \tag{7}$$

Step3: Ec is the center evidence, if it is satisfy with the (8):

$$\delta(E_c) = \max_{1 \le i \le n}(\delta(E_i)) \tag{8}$$

Step4: The weight wi of evidence is defined as:

$$w_i = \delta(E_i) / \delta(E_C) \tag{9}$$

### 3.3 Modified Combination Rules

The weight of the evidence source is wi, wi ∈ [0,1], the combination rules of the weighted basic probability assignment are inheritance from [12], following steps below.

Step1: Calculating all the weights based on algorithm above, the weight vector W = ( w1 , w2 , ⋯, wn )

Step2: According to (10), the probability of the evidence will be re-allocated.

$$m_k^{'}(A_i) = \begin{cases} w_k * m_k(A_i) & A \neq \Theta \\ 1 - \sum_{A_i \neq \phi} w_k * m_k(A_i) & A = \Theta \end{cases} \tag{10}$$

Step3: Calculate the conflicting value k and the average level supporting proposition q(A), which are defined as below.

$$k = \sum_{i,j: A_i \cap A_j = \Phi} m_1^{'}(A_i) m_2^{'}(A_j) \tag{11}$$

$$q(A) = \frac{1}{n} \sum_{i=1}^{n} m_1^{'}(A_i) \tag{12}$$

Step4: Substituting the new probability distribution into the following formula to calculate the synthesis results.

$$\begin{cases} \tilde{m}(A) = \sum_{i,j: A_i \cap A_j = A} m_1^{'}(A_i) m_2^{'}(A_j) + k \bullet q(A) & A \neq \Phi, \Theta \\ \tilde{m}(\Phi) = 0 \\ \tilde{m}(\Theta) = 1 - \sum_{A \in \Theta} \tilde{m}(A) \end{cases} \tag{13}$$

## 4. Experiments Results and Analysis

Through the analysis results of normal and conflict evidences situations, this section will verify the validity of the new method. Suppose $\Theta = \{ a , b , c\}$.

### 4.1 Normal Data

Table 1 Focal Elements Distribution

| Evidence | Propositions | | |
|---|---|---|---|
| | a | b | c |
| m1 | 0.90 | 0 | 0.10 |
| m2 | 0.88 | 0.01 | 0.11 |
| m3 | 0.5 | 0.20 | 0.30 |
| m4 | 0.98 | 0.01 | 0.01 |
| m5 | 0.90 | 0.05 | 0.05 |

Table 1 is the source of evidence for the normal data, there is 5 groups. Seen from Table 1, the artificial reasoning synthesis results should be a.

Table 2 Combination Results

| Algorithm | Evidences | | | |
|---|---|---|---|---|
| | $m_1, m_2$ $K$=0.1970000 | $m_1, m_2, m_3$ $K$=0.6007000 | $m_1, m_2, m_3, m_4$ $K$=0.6118870 | $m_1, m_2, m_3, m_4, m_5$ $K$=0.6507264 |
| Dempster | $m(a)$=0.9863014<br>$m(b)$=0<br>$m(c)$ = 0.0136986<br>$m(\Theta)$ = 0 | $m(a)$=0.9917355<br>$m(b)$=0<br>$m(c)$ = 0.0082645<br>$m(\Theta)$ = 0 | $m(a)$=0.9999150<br>$m(b)$=0<br>$m(c)$ = 0.0000850<br>$m(\Theta)$ = 0 | $m(a)$=0.9999953<br>$m(b)$=0<br>$m(c)$ = 0.0000047<br>$m(\Theta)$ = 0 |
| Yager[13] | $m(a)$=0.7920000<br>$m(b)$=0<br>$m(c)$ = 0.0110000<br>$m(\Theta)$ = 0.1970000 | $m(a)$=0.3960000<br>$m(b)$=0<br>$m(c)$ = 0.0033000<br>$m(\Theta)$ = 0.6007000 | $m(a)$=0.3880800<br>$m(b)$=0<br>$m(c)$ = 0.0000330<br>$m(\Theta)$ = 0.6118870 | $m(a)$=0.3492720<br>$m(b)$=0<br>$m(c)$ = 0.0000017<br>$m(\Theta)$ = 0.6507264 |
| SUN[14] | $m(a)$=0.8930454<br>$m(b)$=0.0005677<br>$m(c)$ = 0.0229211<br>$m(\Theta)$ = 0.0834658 | $m(a)$=0.6591065<br>$m(b)$=0.0242335<br>$m(c)$ = 0.0621528<br>$m(\Theta)$ = 0.2545073 | $m(a)$=0.6754816<br>$m(b)$=0.0193952<br>$m(c)$ = 0.0458762<br>$m(\Theta)$ = 0.2592470 | $m(a)$=0.6612917<br>$m(b)$=0.0202513<br>$m(c)$ = 0.0427544<br>$m(\Theta)$ = 0.2757027 |
| WEI[15] | $m(a)$=0. 96733<br>$m(b)$=0. 0009850002<br>$m(c)$ = 0. 031685<br>$m(\Theta)$ = 0 | $m(a)$=0. 8215649<br>$m(b)$=0. 01044716<br>$m(c)$ = 0. 06840459<br>$m(\Theta)$ = 0. 0995834 | $m(a)$=0. 8478351<br>$m(b)$=0. 006129951<br>$m(c)$ = 0. 03761158<br>$m(\Theta)$ = 0. 1084234 | $m(a)$=0. 8511387<br>$m(b)$=0. 007711634<br>$m(c)$ = 0. 02205105<br>$m(\Theta)$ = 0. 1190986 |

| | | | | |
|---|---|---|---|---|
| PAPER | $K$=0.197<br>$m(a)$=0. 96733<br>$m(b)$=0. 000985<br>$m(c)$ = 0.031685<br>$m(\Theta)$ = 0 | $K$=0.5970444<br>$m(a)$=0. 8523845<br>$m(b)$=0. 04143542<br>$m(c)$ = 0.1043324<br>$m(\Theta)$ = 0.00184768 | $K$=0.60431695<br>$m(a)$=0. 8853891<br>$m(b)$=0. 03264323<br>$m(c)$ = 0. 0777387<br>$m(\Theta)$ = 0. 0042289 | $K$=0.641591966<br>$m(a)$=0. 889182<br>$m(b)$=0. 03395038<br>$m(c)$ = 0. 07212327<br>$m(\Theta)$ = 0. 0047443 |

From table 2, the supporting degree of evidence a increase with the increase of data in normal circumstances, Dempster, Wei[15] and the algorithm are faster speed convergence to a, and get the correct integration results. As evidence increasing, the unknown results of Yager [13], Sun [14] and Wei [15] increase, the uncertain results of the algorithm are far less than the above three algorithms, and close to zero. The overall analysis being seen in the data source of evidence under normal circumstances, Dempster, Wei [15] and the algorithm can get the correct the synthesis results, but our algorithm has the highest reliability.

## 4.2 Conflicting Data

Table 3 Conflicting Data

| Evidence | Propositions | | |
|---|---|---|---|
| | *a* | *b* | *c* |
| $m_1$ | 0.98 | 0.01 | 0.01 |
| $m_2$ | 0 | 0.01 | 0.99 |
| $m_3$ | 0.90 | 0 | 0.10 |
| $m_4$ | 0.90 | 0.01 | 0.10 |

Tables 3 are the conflicting evidences of 4 groups, from the artificial reasoning, when there are only two groups evidences, the synthesis results should be c, but as the number of evidences increases, the end result should be a. evidence 2 is highly conflicts with other evidences.

Table 4 Combination Results

| Algorithm | Evidences | | |
|---|---|---|---|
| | *m1, m2* | *m1, m2, m3* | *m1, m2, m3, m4* |
| *Dempster* | $k$=0.99<br>$m(a)$=0<br>$m(b)$=0.01<br>$m(c)$ = 0.99<br>$m(\Theta)$ = 0 | $k$ = 0.99901<br>$m(a)$= 0<br>$m(b)$=0<br>$m(c)$ = 1<br>$m(\Theta)$ = 0 | $k$ = 0.999901<br>$m(a)$ = 0<br>$m(b)$=0<br>$m(c)$ = 1<br>$m(\Theta)$ = 0 |
| *Yager[13]* | $k$=0.99<br>$m(a)$=0<br>$m(b)$=0.0001<br>$m(c)$ = 0.0099<br>$m(\Theta)$ = 0.99 | $k$ = 0.99901<br>$m(a)$= 0<br>$m(b)$=0<br>$m(c)$ = 0.00099<br>$m(\Theta)$ = 0.99901 | $k$ = 0.999901<br>$m(a)$ = 0<br>$m(b)$=0<br>$m(c)$ = 0.000099<br>$m(\Theta)$ = 0.999901 |

| | | | |
|---|---|---|---|
| *SUN[14]* | *ε= 0.3716*<br>*m(a) = 0.18*<br>*m(b) = 0.004*<br>*m(c) = 0.194*<br>*m(Θ) = 0.622* | *ε= 0.512*<br>*m( a) = 0.321*<br>*m( b) = 0.003*<br>*m(c) = 0.188*<br>*m(Θ) = 0.488* | *ε= 0.604*<br>*m( a) = 0.42*<br>*m( b) = 0.003*<br>*m( b) = 0.181*<br>*m(Θ) = 0.396* |
| *WEI[15]* | *k = 0.99 ,*<br>*m(a) =0.4851,*<br>*m(b) = 0.01*<br>*m(c) =0.5049*<br>*m(Θ) = 0* | *k=0.48251*<br>*m(a)=0.7589*<br>*m(b)= 0.0019*<br>*m(c)=0.1119*<br>*m(Θ) = 0.1272* | *k = 0.44651*<br>*m( a) =0.8341*<br>*m( b) = 0.0012*<br>*m(c) =0.0599*<br>*m(Θ) = 0.1048* |
| *PAPER* | *k = 0.99*<br>*m(a) =0.4851,*<br>*m(b) = 0.01*<br>*m(c) =0.5049 ,*<br>*m(Θ) = 0* | *k = 0.152*<br>*m(a) =0.9337065*<br>*m(b) = 0.0005023976*<br>*m(c)= 0.0145743*<br>*m(Θ) =0.0512168* | *k = 0.099*<br>*m(a) =0.9683049*<br>*m(b) =0.000206358*<br>*m(c) =0.002402914*<br>*m(Θ) =0.02908585* |

From the combination results of tables 4 for the conflict evidences, we can see that Dempster and Yager [13] algorithms do not apply to conflicting evidences combination; Sun [14] algorithm although can be used for the conflict evidences combination, the distribution of precision is not enough, the convergence is slow , the value of uncertain results are high, and as the evidences increasing, the uncertainty value did not significantly reduce, and can not get the recognition results; Wei [15] and our algorithm can get the correct results, and both have reduced the level of conflict, our algorithm fully consider the credibility of evidence and other global information, which greatly reduces the level of conflict between the evidences, and thus minimize the interference of conflicting evidences on the combination results, with a strong anti-interference ability. From Table 4, we can see that the algorithm can have a good decision with three evidences. From the test results, we can also see that, as evidence of supporting a increases, the value of m (a) steadily improves, the uncertainty recognition results are almost zero, and which indicate that our algorithm has high reliability. So, our algorithm can solve the conflict problems in the evidence synthesis, and has the best advantage compared to other algorithms.

## 5. Conclusion

For the combination of conflict evidences, with the combined advantages of evidence pre-processing and modified combination rule, one new algorithm was proposed. By firstly using the Pearson coefficient for the pretreatment the source evidences, and then the modified rules for the combination. Algorithm not only solves the conflict problem of evidence synthesis, but also has high versatility and robustness, and will have some practical value in engineering applications.

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