Quad-rotor UAV Control Method Based on PID Control Law

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Abstract. This paper studies on the dynamics model and control method of quad-rotor UAV for research. Firstly analyzes the dynamic characteristic of quad-rotor UAV and establishes the nonlinear dynamics model of quad-rotor UAV; and then applies the PID control to three channels of pitch, roll and yaw based on the established model, and the simulation results show that PID control is effective for the quad-rotor UAV control with the ideal conditions. Finally analyzes the results of PID control under the circumstance that the data form feedback channel is polluted by noise, which lay the foundation for the improvement of the PID control law.

Keywords: quad-rotor,dynamics model, PID

1. Introduction

Quad-rotor UAV is a kind of butterfly, coaxial type, vertical take-off and landing multi-rotor aircraft. Compared with other types of UAV, it has lots of unique advantages [1]: vertical take-off and landing, hovering, flying in arbitrary direction, adapting to many kinds of takeoff and landing site, etc. Its unique performance makes it have broad application space in military and civilian fields. In the aspect of military, Quad-rotor UAV can be used to perform both tasks of destruction and non-combat missions such as investigation, positioning, communications relay, etc. In the aspect of civil, quad-rotor UAV in monitoring of atmospheric environment, the traffic situation, exploration and resource distribution, detecting electric power line, etc have broad application prospects [2].

Due to quad-rotor UAV movement characteristics present a nonlinear, strong coupling, time-varying, exploring practical flight control method is of great value. Control algorithm design is the core content of quad-rotor UAV flight control, and PID as a classic control algorithm has the advantages of simple structure, easy to implement, which has been widely applied in engineering practice [3]. Therefore, to carry out the quad-rotor UAV control method based on the PID control law is not only theoretical research significance, and lay a foundation for quad-rotor UAV to engineering practical. Aiming at the quad-rotor UAV flight control problem, this paper carry out the research work mainly from the modeling methods, control algorithm and the simulation experiment and other aspects.

2. Foundation of the Dynamic Model of the Quad-Rotor Uav

The motion of UAV can be divided into 6 degrees of freedom, including 3 degrees of freedom of linear motion and 3 degrees of freedom of angle motion around the center of mass[4]. According to Newton’s second law, the dynamic equations of UAV are as follows:

$$\vec{F} = m\frac{d\vec{V}}{dt} \tag{1}$$

$$\vec{M} = \frac{d\vec{H}}{dt} \tag{2}$$

Where $\vec{F}$ represents force acted on the quad-rotor UAV, m is the mass of the UAV, $\vec{V}$ indicates the velocity of the center of mass of the UAC, $\vec{M}$ is resultant moment of force of the UAV and $\vec{H}$ is moment of momentum of UAV relative to the ground coordinates.

2.1 Linear Equation of Motion

The force acted to UAV includes gravity, lift of the rotor and the air resistance.

$$G = mg \tag{3}$$

$$F_i = \frac{1}{2}\rho C_i \omega_i^2 = k_i \omega_i^2 \tag{4}$$

$$D_i = \frac{1}{2}\rho C_d \omega_i^2 = k_d \omega_i^2 \tag{5}$$

Where G is the gravity of the UAV, F_i , $i \in \{1,2,3,4\}$, is the lifting force of the rotor i, D_i is the resistance of the rotor i, C_i is the lift coefficient of the rotor, C_d is the resistance coefficient of the rotor, ω_i is the angular velocity of the rotor i.   is the lift coefficient and k_d is the resistance coefficient.

$\vec{F}$, the resultant lift of the quad rotors, is as follows:

$$\vec{F} = R\begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix} = \begin{bmatrix} \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\sin\theta\cos\phi - \sin\phi\cos\psi \\ \cos\theta\cos\phi \end{bmatrix}\sum_{i=1}^{4} F_i \tag{6}$$

Substitute equation (6) into (1):

$$\begin{cases} \ddot{x} = \left[\left(\text{üüü}\psi\text{üüüü}\theta \quad \phi + \quad \psi \quad \phi\right)\sum_{i=1}^{4} F_i - K_1\dot{x}\right]m^{-1} \\ \ddot{y} = \left[\left(\text{üüü}\psi\text{üüüü}\theta \quad \phi - \quad \phi \quad \psi\right)\sum_{i=1}^{4} F_i - K_2\dot{y}\right]m^{-1} \\ \ddot{z} = \left[\left(\cos\theta\cos\phi\right)\sum_{i=1}^{4} F_i - K_3\dot{z}\right]m^{-1} - g \end{cases} \tag{7}$$

Where (x, y, z) is the location of the center of mass of the UAV, $K_i (i = 1,2,3)$ is the total resistance coefficient and g is gravitational acceleration.

2.2 The Angular Motion Equation

The relation between the angular velocity of ruler angle $(\dot\phi, \dot\theta, \dot\psi)$ and the angular velocity of the fuselage (p, q, r) is as follows:

$$\begin{bmatrix} \dot\theta \\ \dot\phi \\ \dot\psi \end{bmatrix} = \begin{bmatrix} p + p\sin\phi\tan\theta + r\cos\phi\tan\theta \\ q\cos\phi - r\sin\phi \\ q\sin\phi\sec\theta + r\cos\phi\sec\theta \end{bmatrix} \tag{8}$$

Assuming that the shape of the quad-rotor UAV is symmetrical, the mass distribution is uniform, it can be assumed that the center of gravity of the UAV is located at the center of the quad-rotor UAV. According to the calculation of the angular momentum, the motion equation of the angle of the quad-rotor UAV is as follows:

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \dot p I_x - \dot r I_{xz} + qr(I_z - I_y) - pqI_{xz} \\ \dot q I_y - pr(I_x - I_z) + (p^2 - r^2)I_{xz} \\ \dot r I_z - \dot p I_{xz} + pq(I_y - I_x) + qrI_{xz} \end{bmatrix} \tag{9}$$

Where M_x, M_y, M_z respectively represents the component of the resultant external angular momentum of the UAV around the axis of X, Y, Z .

According to equation (9):

$$\begin{bmatrix} \dot p \\ \dot q \\ \dot r \end{bmatrix} = \begin{bmatrix} \left[M_x + \left(I_x - I_z\right)qr\right] / I_x \\ \left[M_y + \left(I_z - I_x\right)pr\right] / I_y \\ \left[M_z + \left(I_x - I_y\right)pr\right] / I_z \end{bmatrix} \tag{10}$$

Finishing equation (9) and equation (10):

$$\begin{cases} \ddot\phi = \left[M_x - \dot\theta\dot\psi\left(I_z - I_y\right)\right] / I_x \\ \ddot\theta = \left[M_y - \dot\phi\dot\psi\left(I_x - I_z\right)\right] / I_y \\ \ddot\psi = \left[M_z - \dot\phi\dot\theta\left(I_y - I_x\right)\right] / I_z \end{cases} \tag{11}$$

According to the ‘X’ flight model of UAV and theorem of momentum:

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} l\left(F_1 + F_4 - F_2 - F_3\right) \\ l\left(F_1 + F_3 - F_2 - F_4\right) \\ \lambda\left(F_1 + F_2 - F_3 - F_4\right) \end{bmatrix} \tag{12}$$

Where l is the distance between the motor shaft and the center of mass of the UAV, λ is the coefficient which link the lift and the twisting moment.

Assuming the input of the system of the quad-rotor UAV as follows:

$$U=\begin{bmatrix}U_1\\U_2\\U_3\\U_4\end{bmatrix}=\begin{bmatrix}F_1+F_2+F_3+F_4\\F_1+F_4-F_2-F_3\\F_1+F_3-F_2-F_4\\F_1+F_2-F_3-F_4\end{bmatrix} \tag{13}$$

Where U_1 is the vertical controlled quantity, U_2 is the roll controlled quantity, U_3 is the pitch controlled quantity and U_4 is the yaw controlled quantity.

Integrate equation (11), (12), (13):

$$\begin{cases}\ddot{\phi}=U_2 l I_x^{-1}\\\ddot{\theta}=U_3 l I_y^{-1}\\\ddot{\psi}=U_4\lambda I_z^{-1}\end{cases} \tag{14}$$

Assuming that the flight environment of the quad-rotor UAV is indoor or breeze outside, the resistance coefficient in equation (7), $K_i(i=1,2,3)$ will be negligible. Then integrate equation (14) and equation (7):

$$\begin{cases}\ddot{x}=\left[(\cos\psi\sin\theta\cos\phi+\sin\psi\sin\phi)U_1\right]m^{-1}\\\ddot{y}=\left[(\sin\psi\sin\theta\cos\phi-\sin\phi\cos\psi)U_1\right]m^{-1}\\\ddot{z}=\left[(\cos\theta\cos\phi)U_1\right]m^{-1}-g\\\ddot{\phi}=U_2 l I_x^{-1}\\\ddot{\theta}=U_3 l I_y^{-1}\\\ddot{\psi}=U_4\lambda I_z^{-1}\end{cases} \tag{15}$$

3. The Control Method of the Quad-Rotor UAV Based on PID Control Law

The principal structure of the PID controller is as shown in Figure 1[5].

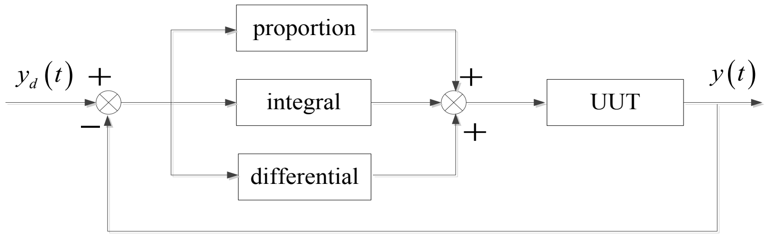

Figure.1 Program block diagram of PID

Where $y_d(t)$ is set point, $y(t)$ is the real output of system. PID controller use the error value between $y_d(t)$ and $y(t)$.

$$e(t)=y_d(t)-y(t) \tag{16}$$

The control law is as follows:

$$u(t)=k_p\left(e(t)\right)+k_i\int_0^t e(t)\,\mathrm{d}t+k_d\,\frac{\mathrm{d}\left(e(t)\right)}{\mathrm{d}t} \tag{17}$$

Where k_p is proportionality coefficient, k_i is integral coefficient and k_d is differential coefficient.

Exert PID control to the pitch, yaw, roll channel respectively. The structure drawing of PID control of the roll channel is as shown in the Figure 2 [6].

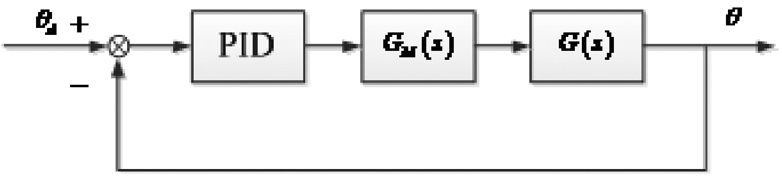

Figure.2 PID control flow chart of Roll

Where $G_M(s)$ is the transfer function of the electrical machine and $G(s)$ is the transfer function of the quad rotor UAV.

The step response exerting PID control to the pitch, yaw, roll channel respectively is as shown in the Figure3, 4, 5. And the control quality is as shown in the table 1.

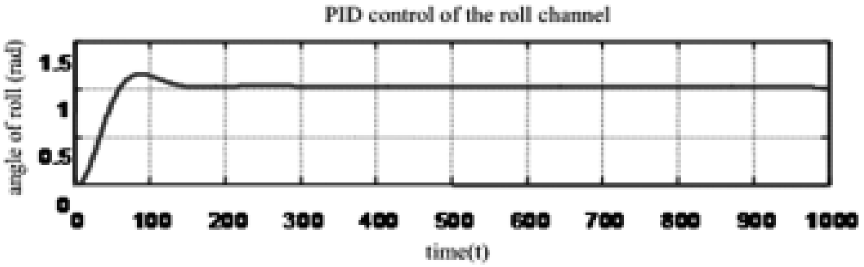

Figure.3 PID control of the roll channel

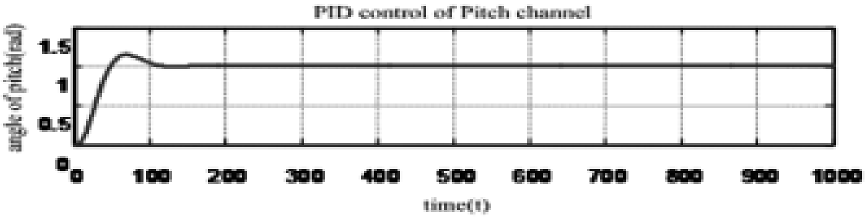

Figure.4 PID control of the pitch channel

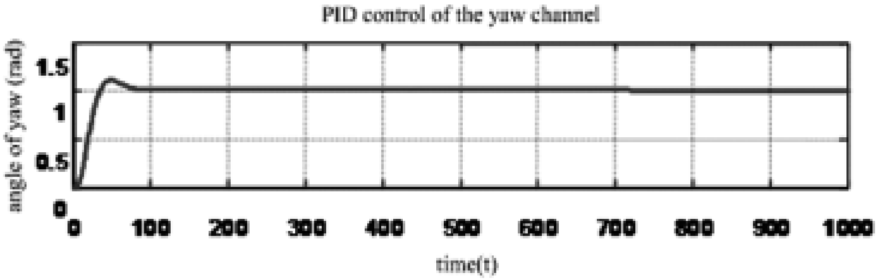

Figure.5 PID control of the yaw channel

Table 1 PID control quality

channel	k_p	k_i	k_d	risetime	overshoot
Roll	0.89	1.87×10^{-2}	0.073	0.38	15.4%
Pitch	0.73	1.45×10^{-2}	0.079	0.29	15.3%
Yaw	0.12	4.5×10^{-2}	0.078	0.21	11.3%

According to the response of the three channel and control quality, PID control method is efficient to control the quad-rotor UAV under the ideal environment.

But in the real flight, the sensor data of the feedback channel are easily polluted by measurement noise. The response of the roll channel polluted by the noise is shown in the figure 6. It is necessary to study the control method under the condition that the sensor data are polluted by noise.

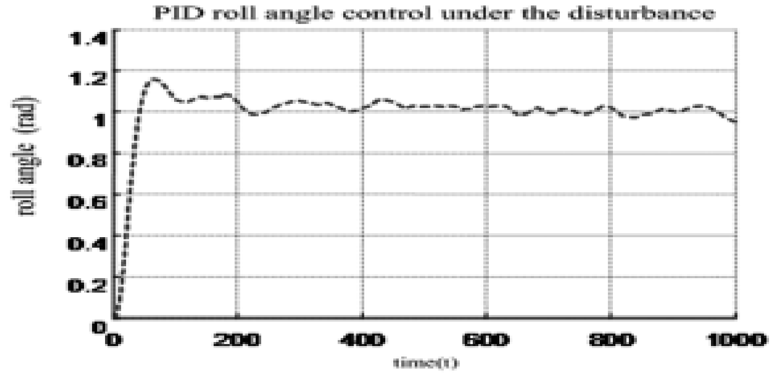

Figure.6 PID control result with noise interference

4. Conclusion

In this paper, the dynamic model of quad-rotor UAV is established, and the PID controllers of the pitch, yaw and roll channels are designed. The simulations reveal that PID control method is efficient to control the quad -rotor UAV under the ideal environment. Finally, taking the roll channel as an example to analysis the result of PID control with the noise disturbance in sensor data, laying the foundation to study the new control method under the condition that the sensor data are polluted by noise.

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